Seat No. : ______________

AC-114

April -2018

B.Sc., Sem.-VI

CC-308 : Mathematics
(Analysis – II)

Time : 3 Hours] **[Max. Marks : 70**

Note : (1) Each question is *compulsory*.
(2) Figures to the right indicates full marks to the question.

1. (a) Let g be continuous on [a, b] and f has a derivatives which is continuous and never changes sign on [a, b], then for some $c \in [a, b]$ prove that

$$\int_a^b f(x)\, g(x)\, dx = f(a) \int_a^c g(x)\, dx + f(b) \int_c^b g(x)\, dx.$$ **7**

OR

If f, g ∈ R [a, b] and g is bounded away from zero, then prove that f/g ∈ R [a, b].

(b) If P_1 and P_2 are two partitions of [a, b], then prove that $U[f; P_1] \geq L[f, P_2]$. **7**

OR

Let $f(x) = \frac{x^2}{3}$ · For each $n \in N$ σ_n be the partition $\left\{0, \frac{1}{n}, \frac{2}{n}, \ldots\ldots \frac{n}{n}\right\}$ of [0, 1], then compute $\lim\limits_{n \to \infty} U[f; \sigma_n]$ and $\lim\limits_{n \to \infty} L[f; \sigma_n]$.

2. (a) State and prove Cauchy's condensation test and hence prove that the series $\sum\limits_{n=2}^{\infty} \frac{1}{n (\log n)^2}$ converges· **7**

OR

Let Σa_n be a divergent series of positive numbers, then prove that there is a sequence (ϵ_n) of positive numbers which converges to zero but $\sum\limits_{n=1}^{\infty} \epsilon_n\, a_n$ diverges.

(b) State and prove the limit form of the comparison test for the convergence of the series. Discuss the convergence of $\sum_{n=1}^{\infty} \frac{1+n}{2+3n^{3/2}}$. **7**

OR

Prove that the absolute convergent series is convergent. Is converse true ? Give support to you answer.

3. (a) If Σa_n is absolutely convergent, then prove that any rearrangement of Σa_n has the same sum. **7**

OR

If $\Sigma a_n z_0^n$ is convergent, then prove that $\Sigma a_n z^n$ is absolutely convergent for $|z| < |z_0|$. Also, discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{z^n}{(n+1)^\alpha}$ $(\alpha \in R)$.

(b) State and prove Merten's theorem. **7**

OR

Prove that the series $\sum_{n=0}^{\infty} (-1)^n \frac{1}{\sqrt{n+1}}$ is conditionally convergent but its Cauchy product with itself is divergent.

4. (a) Obtain the Maclaurin series expansion of cos x for $-\infty < x < \infty$. Also, obtain series for cos (0.1). **7**

OR

Write down Taylor's formula with Lagrange's form of remainder for $f(x) = \log(1 + x)$ about a = 2 and n = 4.

(b) Obtain the power series solution of the differential equation $y'' + y = 0$ with the condition $y(0) = 0$, $y'(0) = 1$. **7**

OR

State and prove Binomial series theorem.

5. Attempt any **seven** : **14**

(1) Find the primitive F of $f(x) = 4 \cos x + e^{x}$.

(2) Evaluate : $\int_{0}^{5} [x] \, dx$.

(3) Discuss the absolute convergence of $\sum_{n=2}^{\infty} \frac{(-1)^n}{\log n}$.

(4) Test the convergence :

$$2 + 2^{1/2} + 2^{1/3} + 2^{1/4} + \ldots$$

(5) Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{n^3 x^n}{n!}$.

(6) Define Cauchy product of the series.

(7) State Maclaurin's Theorem with Cauchy Form of Remainder.

(8) State the series of $\sin x$, for any real x.

(9) Test the convergence of $\int_{0}^{\infty} \frac{1}{e^x} \, dx$.